A probabilistic approach for the fatigue life prediction of structures

Martin Poncelet[a], Cédric Doudard[b], S. Calloch[b], B. Weber[c], A. Galtier[d], F. Hild[a]

[a] LMT-ENS Cachan, Cachan, France
[b] LMSN-ENSIETA, Brest, France
[c] ARCELOR RESEARCH, Meziere-les-Metz, France
[d] CETIM, Senlis, France

Abstract The goal of this study is to predict quickly the fatigue limit of a structure under multiaxial loadings thanks to self-heating tests results. A probabilistic model linking self-heating tests and high cycle fatigue results is thus extended to heterogeneous proportional and non-proportional loadings by use of an effective dissipative stress. A way to identify the model parameter is to apply it to a small structure. Tubular tension-torsion specimens made of C45 (AISI 1045) steel are chosen. Fatigue limits for different loadings are estimated after identification of the parameters of the model on self-heating results and a fatigue limit in tension. Predictions of fatigue limit and experimental results are in good agreement.

1 INTRODUCTION

Engineering structures are often subjected to multiaxial loadings. In the same time, these structures are most of the time design by taking only uniaxial loadings into account because few experimental results concerning multiaxial experiments are available. In order to better predictions of fatigue life of structures, an alternative way can be used : instead of performing traditionnal time-consuming methods (e.g., staircase tests) to evaluate the scatter of the fatigue results under multiaxial loadings, one can predict rapidly the fatigue limit thanks to self-heating tests.

The self-heating test (or main fast estimation method), is based on the idea that the temperature of the specimen under cyclic loading is linked to it's fatigue limit. One measures the change of the specimen temperature under uniaxial cyclic loadings with thermocouples [1,2] or thermography [3, 4] and then plots the steady-state temperature as a function of the stress amplitude applied to the specimen (Fig. 1). For some materials such as steel or cast iron, a first part of the curve shows nearly no change in temperature, whereas in the second part the temperature increases significantly with the stress amplitude. A correlation between the mean fatigue limit and the stress level leading to the temperature increase has been empirically proposed. The assumption of a dissipation induced by microplasticity explains this correlation. A probabilistic approach based on a Poisson point process controlling microplasticity distributions even relates the thermal results to the scatter of fatigue results (i.e., microplasticity leads to microcrack inception, and thus to failure) [2, 5].

This method has already been applied to samples, but never fully extended to structures under proportional and non-proportional heterogeneous stress states. The present study aims to prove that the prediction of such a structure is still possible thanks to an adequat multiaxial HCF criterion identified on self-heating tests results. The method consists in measuring temperatures for different proportional loading-paths in a given stress plane (tension-torsion plane in the present case) [6, 7] and then apply a data processing taking into account the heterogeneity of the stress field by using an extension of the concept of effective volume [8]. The validation of this model is based on the HCF life prediction under torsion loading, in and out-of-phase tension-torsion loadings.

2 HETEROGENEOUS PROBABILISTIC MODEL

2.1 Modelisation of microplasticity onset

HCF damage is supposed to be localized at the microscopic scale and induced by microplastic activity (in the grains whose orientation is favorable). Microplasticity is modeled at the scale of slip bands by Schmid's law. One direction becomes active when the shear stress amplitude T_a is greater than the critical shear τ_y, which is assumed to be scattered. It is considered that the probability of finding k active directions within a domain Ω of volume V follows a Poisson distribution [9, 10]

$$P_k(\Omega) = \frac{[-N(\Omega)]^k}{k!}\exp[-N(\Omega)] \tag{1}$$

where $N(\Omega)$ is equal to the average number directions, which is related to the intensity of the process λ (i.e., the average density of active directions)

$$N(\Omega) = \lambda\, V\ . \tag{2}$$

It is proposed that the intensity of the process follows a power law of the macroscopic shear amplitude integrated over all directions in space (defined by the solid angle Θ) [8]

$$\lambda(\Sigma_a) = \frac{1}{V_0\left(S_0 + \alpha I_{1,max}\right)^m}\int (2T_a)^m\, d\Theta \tag{3}$$

where m, α and $V_0(S_0)^m$ are three parameters depending on the considered material, and $I_{1,max}$ the maximum hydrostatic stress over a given cycle. The shear stress τ for the considered direction is related to the macroscopic shear stress T and the plastic slip γ^p by [11, 8]

$$\tau = T - \mu(1-\beta)\gamma^p \tag{4}$$

with $\beta = \dfrac{2\,(4-5\nu)}{15\,(1-\nu)}$ given by the Eshelby analysis of a spherical inclusion, μ and ν the shear modulus and Poisson's ratio, respectively [12]. No hardening is considered for the study at the scale of the slip direction (it is sufficient to obtain a non linear kinematic hardening at the mesoscopic scale).

The intrinsic dissipated energy in one site of volume V_0 during a cycle for a given critical shear τ_y and a shear amplitude T_a for the site's direction is expressed as [8]

$$D_{site}\left(T_a, \tau_y\right) = \frac{4V_0}{h'}\tau_y\left\langle T_a - \tau_y\right\rangle \tag{5}$$

where $h'=\mu(1-\beta)$ and $\langle\,.\,\rangle$ are the Macauley brackets (i.e., positive part of '.'). With a Poisson point process and for a given direction, $\dfrac{d\lambda}{dT}V\,dT$ corresponds to the mean number of sites activated for a shear amplitude ranging from T and (T + dT) in a domain of volume V, i.e., the number of sites whose critical shear stress varies between T and (T + dT), and whose dissipated energy during a load cycle is $D_{site}(T_a, T)$. The global dissipated energy density Δ during a cycle reads

$$\Delta V = \int_\Omega\int_0^{T_a} D_{site}(T_a, T)\frac{d\lambda}{dT}dT dV = \int_\Omega\int_0^{T_a}\frac{V_0}{h'}\left\langle T_a - T\right\rangle T\frac{4m\int (2T)^{m-1}d\Theta}{V_0(S_0+\alpha I_{1,max})^m}dT dV\ . \tag{6}$$

2.2 Introduction of heterogeneity factor

The former equation is related to Σ_T, the Tresca equivalent stress amplitude, by

$$\Delta = \frac{4 V_0 m}{h'(m+1)(m+2)} \frac{\left(\Sigma_{FT}\, \tilde{H}_{m+2}\right)^{m+2}}{\left(V_0^{1/m}\left(S_0 + \alpha I_{1,max}\right)\right)^m} \tag{7}$$

where

$$\Sigma_{FT} = \underset{\Omega}{\mathrm{Max}}\,(\Sigma_T)\,, \tag{8}$$

and the heterogeneity factor is defined by [13, 8]

$$\tilde{H}_{m+2} = \frac{1}{V} \int_\Omega \left(\frac{\Sigma_T}{\Sigma_{FT}}\right)^{m+2} \kappa(m+2) dV\,, \tag{9}$$

where $\kappa(m)$ represents the distribution of activated directions

$$\kappa(m+2) = \int \left(\frac{2T_a}{\Sigma_T}\right)^{m+2} d\Theta \tag{10}$$

The factor $\tilde{H}_m$ thus able to take into account the heterogeneity of the loading (integration over the total considerated volume) and the multiaxiality of the loading thanks to $\kappa(m)$ which depends on the direction of the loading and/or the non-proportionnality of it.

2.3 Thermal behaviour under cyclic loadings

To describe the thermal effects under cyclic loadings, the heat conduction equation is considered. By integration over the central part of the specimen the mean temperature of this part is related to the mean microplastic dissipation by

$$\dot{\theta} + \frac{\theta}{\tau_{eq}} = \frac{\Delta f_r}{\rho c} \tag{11}$$

where f_r is the load frequency, τ_{eq} a characteristic time that depends on the heat transfer boundary conditions, ρ the mass density and c the specific heat capacity [14, 15]. Consequently, the steady-state temperature is given by

$$\bar{\theta} = \frac{\eta m}{(m+1)(m+2)} \frac{\left(\Sigma_{FT}\, \tilde{H}_{m+2}\right)^{m+2}}{\left(V_0^{1/m}\left(S_0 + \alpha I_{1,max}\right)\right)^m} \tag{12}$$

where $\eta = \dfrac{4 V_0 f_r \tau_{eq}}{h'\rho c}$. In the next section, it is proposed to use the same framework to model HCF results.

2.4 HCF life

To describe HCF results, the weakest link theory is considered. The failure probability is then given by the probability of finding at least one active direction. By using Eqs. (1-3), the failure probability is related to the shear amplitude

$$P_F = 1 - \exp\left[-\frac{1}{V_0} \iint_{\Omega} \left(\frac{2T_a}{S_0 + \alpha I_{1,max}} \right)^m d\Theta dV \right] \quad (13)$$

By using Eq. (10), it is expressed as

$$P_F = 1 - \exp\left[-\frac{V\tilde{H}_m}{V_0} \left(\frac{\Sigma_{FT}}{S_0 + \alpha I_{1,max}} \right)^m \right] . \quad (14)$$

Equation (14) corresponds to a Weibull model [16]. With this approach, the mean fatigue limit (i.e., the Tresca equivalent stress amplitude) and the coefficient of variation CV are given by

$$\overline{\Sigma}_\infty = (S_0 + \alpha I_{1,max}) \left(\frac{V_0}{V\tilde{H}_m} \right)^{\frac{1}{m}} \Gamma\left(1 + \frac{1}{m}\right) \quad (15)$$

$$CV = \frac{\sqrt{\Gamma\left[1 + \frac{2}{m}\right] - \Gamma^2\left[1 + \frac{1}{m}\right]}}{\Gamma\left(1 + \frac{1}{m}\right)} \quad (16)$$

where $\Gamma(m) = \int_0^{+\infty} x^{m-1} \exp(-x) dx$ is the gamma function. Equation (15) accounts for the influence of several parameters on fatigue properties, namely, the stress heterogeneity, the volume effect, the hydrostatic stress and the multiaxiality of the loading with the parameter $\kappa(m)$ as shown in Ref. [8].

To model the S-N curve for a multiaxial test, it is assumed that microcrack initiation is related to a critical value of the dissipated energy E_c

$$N = \frac{A'}{\langle \Sigma_T - \Sigma_\infty(P_F) \rangle \Sigma_\infty(P_F)} \quad (17)$$

where $A' = \frac{h' E_c}{V_0}$. Equation (17) corresponds to Stromeyer's law [17]. The present model depends on three parameters m, α and $V_0(S_0)^m$ to predict the fatigue limits. Another parameter A' is needed to describe the S-N curve and a last one η to account for thermal effects under cyclic loadings. In the next section, it is proposed to apply the proposed model.

3 APPLICATION TO TENSION-TORSION LOADINGS

Tubular specimens machined from the same bar of medium carbon steel C45 (SAE45) are used for tension-torsion loadings. The external and internal radii are respectively R_e=7 mm and R_i=5.5 mm. Differential temperature measurements are obtained by use thermocouples. At a loading frequency of 5Hz, 3,000 cycles are needed for each step to reach steady state conditions. The macroscopic stress tensor that depends on the radius r is defined by

$$\Sigma = \begin{pmatrix} \Sigma_{max} \sin(2\pi f_r t) + \Sigma_m & T_{max} \sin(2\pi f_r t + \delta) & 0 \\ T_{max} \sin(2\pi f_r t + \delta) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (18)$$

where Σ_{max} and $T_{max}=\tan(\phi)\, \Sigma_{max}$ are the tensile and shear stress amplitudes, δ the phase difference between shear and normal stresses, and Σ_m the mean tension stress.

To apply this model, a particular attention must be paid to the determination of the stress heterogeneity factor [7, 8]. The values of this factor for different loading paths are given in Table. I.

Table I. Values of $\tilde{H}_m$ for different loading path (m=12 and Σ_m=0)

Loading	Prop. (ϕ=0°)	Prop. (ϕ=90°)	Prop. (ϕ=45°)	N-Prop. (ϕ=45°, δ=π/2)
$\tilde{H}_m$	4.8	0.6	1.4	3.1

The parameters m and η are obtained by using the change of the steady state temperature with the Tresca equivalent stress amplitude for ϕ=90° and Σ_m=0 (Fig. 1a). The parameter α is determined from self-heating test results for ϕ=0° and Σ_m=0 (Fig. 1b). The last parameter $V_0(S_0)^m$ is obtained by using Eq. (15) of the mean fatigue limit determined by staircase tests for ϕ=0° and Σ_m=0.

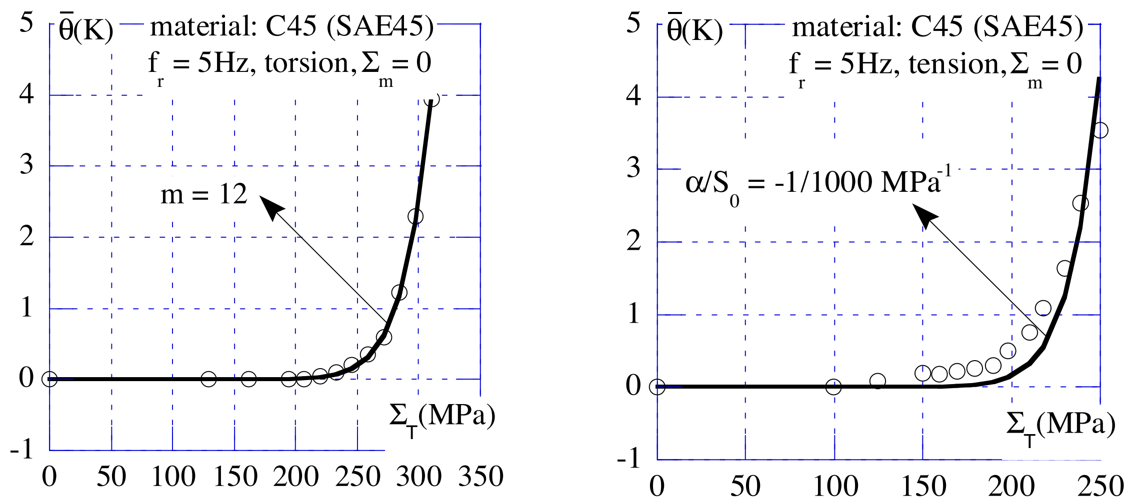

FIGURE: 1 – Identification of the parameters m, α and η from the thermal effects for two loading paths: torsion (left) and tension (right).

4 VALIDATION OF THE MODEL

The first validation concerns the prediction of the self-heating test results for in and out-of phase tension-torsion loadings. Figure 2 shows the change of the steady-state temperature as function of an effective dissipated stress amplitude defined by

$$\Sigma_{eff\ diss} = \frac{\Sigma_{FT}\left(\tilde{H}_{m+2}\right)^{1/(m+2)}}{\left(1+\alpha I_{1,max}/S_0\right)^{m/(m+2)}} . \tag{19}$$

The thermal effects are predicted in a reasonable way.

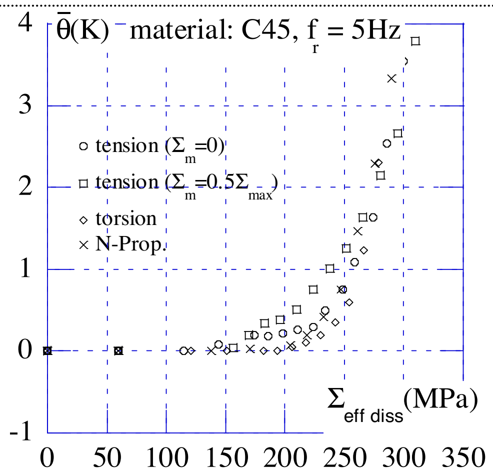

FIGURE: 2 - Steady-state temperature for different loading paths: tension with Σ_m=0, tension with $\Sigma_m=0.5\times\Sigma_{max}$, torsion and non-proportional (ϕ=48°, $\delta=\pi/2$) as functions of the effective dissipated stress amplitude.

The second validation concerns the prediction of the mean fatigue limit for in and out-of phase tension-torsion loadings. The relative error of prediction (REP) is defined by

$$\mathrm{REP} = \frac{\overline{\Sigma}_\infty^{\exp} - \overline{\Sigma}_\infty^{\mathrm{pre}}}{\overline{\Sigma}_\infty^{\exp}} \tag{20}$$

Table II shows that the predictions are in agreement with experimental results obtained with 3 staircases of 7 samples each.

TABLE 2 - Values of REP for different loading path

Loading	Prop. (ϕ=90°)	Prop. (ϕ=45°)	N-Prop. (ϕ=45°, $\delta=\pi/2$)
REP	10%	7%	-5%

5 CONCLUSION

Self-heating tests and fatigue tests under proportional and non-proportional loading have been performed on tubular specimens. The proposed probabilistic approach is able to relate thermal effects to fatigue properties under multiaxial cyclic loadings. The present model accounts for the influence of the stress heterogeneity [4], the volume effect and the hydrostatic stress on fatigue properties, but also the influence of multiaxial loadings [8]. The major interest of this method is it rapidity because self-heating tests are much shorter than traditional method (e.g. staircase tests). The failure probability and the thermal effects are well predicted by the model for different effective volumes, and for different multiaxial loadings (in and out-of phase tension-torsion loadings). However this approach has been applied here to a very simple structure (a tube) to identify the different parameters of the model, it proves that it is appliable, apriori, to any structure so long as its stress heterogeneity is known.

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